List of Probability Density Function and Cumulative Distribution Function for Common Continuous Random Variable

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1. Normal Distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad (\sigma > 0)$$

$$F(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

2. Log-Normal Distribution

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \quad (\sigma > 0)$$

$$F(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\ln x - \mu}{\sigma\sqrt{2}}\right)\right]$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

3. Skew-Normal Distribution

$$f(x) = \frac{2}{\omega\sqrt{2\pi}} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (\omega > 0)$$

$$F(x) = \Phi\left(\frac{x-\xi}{\omega}\right) - 2T\left(\frac{x-\xi}{\omega}, \alpha\right)$$

with $\Phi(x) = \int_{-\infty}^{x} \phi(t) dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$ and $T(h, a) = \frac{1}{2\pi} \int_0^a \frac{e^{-\frac{1}{2}h^2\left(1+x^2\right)}}{1+x^2} dx \quad (-\infty < h, a < +\infty)$ is Owen's T function.

4. Alpha-Skew-Normal Distribution

$$f(x) = \frac{(1-\alpha x)^2 + 1}{2 + \alpha^2} \phi(x)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are p.d.f. and c.d.f. of the normal distribution with mean μ and variance σ^2.

$$F(x) = \Phi(x) + \alpha\left(\frac{2-\alpha x}{2+\alpha^2}\right)\phi(x)$$

5. Log-Skew-Normal Distribution

$$f(x) = \frac{2}{x}\phi(\ln x)\Phi(\lambda \ln x)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are p.d.f. and c.d.f. of the normal distribution with mean μ and variance σ^2.

6. Generalized-Normal Distribution

$$f(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)}e^{-(|x-\mu|/\alpha)^\beta} \quad (\alpha > 0; \beta > 0)$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ denotes the gamma function.

$$F(x) = \frac{1}{2} + \text{sign}(x-\mu)\frac{1}{2\Gamma(1/\beta)}\gamma\left(1/\beta, \left|\frac{x-\mu}{\alpha}\right|^\beta\right)$$

where β is a shape parameter, α is a scale parameter and $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt$ is the unnormalized incomplete lower gamma function.

7. Mixture of Two Normal

$$f(x,p) = pf_1(x) + (1-p)f_2(x)$$

with $0 < p < 1$, for $i = 1,2$

$$f_i(x) = \frac{1}{\sigma_i\sqrt{2\pi}}e^{-(x-\mu_i)^2/2\sigma_i^2}$$

8. Beta-Normal Distribution

$$F(x) = G\left[\Phi\left(\frac{x-\mu}{\sigma}\right)\right] = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left[\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\alpha-1} \times \left[1-\Phi\left(\frac{x-\mu}{\sigma}\right)\right]^{\beta-1}\sigma^{-1}\phi\left(\frac{x-\mu}{\sigma}\right)$$

where

$$G(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^{\Phi(x)} t^{\alpha-1}(1-t)^{\beta-1}dt, \quad 0 < \alpha, \quad \beta < \infty$$

and

$$g(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\Phi(x)^{\alpha-1}(1-\Phi(x))^{\beta-1}\Phi'(x)$$

and $\Phi(x)$ is the c.d.f. of normal distribution with mean μ and variance σ^2.

9. Arcsine Gaussian Distribution

$$f(x) = \frac{\phi(x)}{\pi\sqrt{\Phi(x)[1-\Phi(x)]}}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are p.d.f. and c.d.f. of the normal distribution with mean μ and variance σ^2.

10. Double Gaussian (Two-Piece Normal) Distribution

$$f(x) = \begin{cases} A\exp\left[-(x-\mu)^2/2\sigma_1^2\right], & x \le \mu, \\ A\exp\left[-(x-\mu)^2/2\sigma_2^2\right], & x \ge \mu, \end{cases}$$

where $A = \left(\sqrt{2\pi}\,(\sigma_1+\sigma_2)/2\right)^{-1}$.

11. Inverse Gaussian Distribution

$$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}}\exp\left[-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]$$

$$F(x) = \Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu}-1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu}+1\right)\right)$$

and $\Phi(x)$ is the c.d.f. of standard normal distribution.

12. Laplace Distribution

$$f(x) = \frac{1}{2\sigma}\exp\left(-\frac{|x-\mu|}{\sigma}\right) \quad (\sigma > 0)$$

$$F(x) = \begin{cases} \frac{1}{2}\exp\left(\frac{x-\mu}{\sigma}\right) & \text{if } x \le \mu \\ 1-\frac{1}{2}\exp\left(-\frac{x-\mu}{\sigma}\right) & \text{if } x \ge \mu \end{cases}$$

13. Beta Laplace Distribution

$$f(x) = \begin{cases} \{2^a B(a,b)\}^{-1}\exp(-|x|)\exp\{-|x|(a-1)\}\{1-\exp(-|x|)/2\}^{b-1}, & x < 0, \\ \{2^b B(a,b)\}^{-1}\exp(-|x|)\exp\{-|x|(b-1)\}\{1-\exp(-|x|)/2\}^{a-1}, & x \ge 0 \end{cases}$$

where $a > 0$ and $b > 0$

$$F(x) = \begin{cases} I_{\exp(x)/2}(a,b), & x < 0, \\ I_{1-\exp(-x)/2}(a,b), & x \ge 0. \end{cases}$$

14. Double Exponential (Skew Laplace) Distribution

$$f(x) = \begin{cases} \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_1+\sigma_2}e^{-\frac{x^2}{2\sigma_2^2}}, & x < 0 \\ \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_1+\sigma_2}e^{-\frac{x^2}{2\sigma_1^2}}, & x \ge 0, \end{cases}$$

where $\sigma_1 > 0$ and $\sigma_2 > 0$.

15. Asymmetric Laplace Distribution

$$f(x) = \frac{\lambda}{\kappa + 1/\kappa}\begin{cases} \exp((\lambda/\kappa)(x-\mu)) & \text{if } x < \mu \\ \exp(-\lambda\kappa(x-\mu)) & \text{if } x \ge \mu \end{cases}$$

where, μ is a location parameter, $\lambda > 0$ is a scale parameter, and κ is an asymmetry parameter.

$$F(x) = \begin{cases} \frac{\kappa^2}{1+\kappa^2}\exp((\lambda/\kappa)(x-\mu)) & \text{if } x \le \mu \\ 1 - \frac{1}{1+\kappa^2}\exp(-\lambda\kappa(x-\mu)) & \text{if } x > \mu \end{cases}$$

16. Exponential Distribution

$$f(x) = \lambda e^{-\lambda x} \quad (\lambda > 0)$$

$$F(x) = 1 - e^{-\lambda x}$$

17. Gamma Distribution

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad (\alpha > 0; \beta > 0)$$

$$F(x) = \frac{1}{\Gamma(\alpha)}\gamma(\alpha, \beta x)$$

where $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the unnormalized incomplete lower gamma function.

18. Beta Distribution

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha, \beta)} \quad (\alpha > 0; \beta > 0)$$

where $\mathrm{B}(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and Γ is the Gamma function.

$$F(x) = \frac{\mathrm{B}(x; \alpha, \beta)}{\mathrm{B}(\alpha, \beta)}$$

where $\mathrm{B}(x; a, b) = \int_0^x t^{a-1}(1-t)^{b-1} dt$ is the incomplete beta function.

19. Beta Prime Distribution

$$f(x) = \frac{(x/\sigma)^{\alpha-1}(1 + x/\sigma)^{-\alpha-\beta}}{\mathrm{B}(\alpha, \beta)\sigma} \quad (\alpha > 0; \beta > 0)$$

$$F(x) = I_{\frac{x/\sigma}{1+x/\sigma}}(\alpha, \beta)$$

where $I_x(\alpha, \beta) = \frac{\mathrm{B}(x;a,b)}{\mathrm{B}(a,b)}$ is the incomplete beta function.

20. Logistic Distribution

$$f(x) = \frac{e^{-(x-\mu)/s}}{s\left(1 + e^{-(x-\mu)/s}\right)^2} \quad (s > 0)$$

$$F(x) = \frac{1}{1 + e^{-(x-\mu)/s}}$$

21. Double Logistic Distribution

$$f(x) = \begin{cases} \frac{1}{\sigma_1+\sigma_2}\frac{2e^{\frac{x}{\sigma_2}}}{\left(1+e^{\frac{x}{\sigma_2}}\right)^2}, & x < 0 \\ \frac{1}{\sigma_1+\sigma_2}\frac{2e^{-\frac{x}{\sigma_1}}}{\left(1+e^{-\frac{x}{\sigma_1}}\right)^2}, & x \ge 0, \end{cases}$$

where $\sigma_1 > 0$ and $\sigma_2 > 0$.

22. Generalized Extreme Value Distribution

$$f(s) = \begin{cases} \exp(-s)\exp(-\exp(-s)) & \text{for } \xi = 0 \\ (1+\xi s)^{-(1+1/\xi)} \exp\left(-(1+\xi s)^{-1/\xi}\right) & \text{for } \xi \neq 0 \text{ and } \xi s > -1 \\ 0 & \text{otherwise} \end{cases}$$

where $s = (x-\mu)/\sigma$ and $\sigma > 0$.

$$F(s) = \begin{cases} \exp(-\exp(-s)) & \text{for } \xi = 0 \\ \exp\left(-(1+\xi s)^{-1/\xi}\right) & \text{for } \xi \neq 0 \text{ and } \xi s > -1 \\ 0 & \text{for } \xi > 0 \text{ and } \xi s \leq -1 \\ 1 & \text{for } \xi < 0 \text{ and } \xi s \leq -1 \end{cases}$$

23. Weibull Distribution

$$f(x) = \frac{k}{\sigma}\left(\frac{x}{\sigma}\right)^{k-1} e^{-(x/\sigma)^k} \quad (x > 0; k > 0; \sigma > 0)$$

$$F(x) = 1 - e^{-(x/\sigma)^k}$$

24. Gumbel Distribution

$$f(x) = \frac{1}{\sigma} e^{-\left(z+e^{-z}\right)}$$

where $z = \frac{x-\mu}{\sigma}$

$$F(x) = e^{-e^{-(x-\mu)/\sigma}}$$

25. Student's t Distribution

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad (\nu > 0)$$

$$F(x) = \frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \times \frac{{}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}$$

where ${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$ is the hypergeometric function.

26. Skew t Distribution

$$f(x) = 2g(x;v)G\left(\lambda x\sqrt{\frac{1+v}{x^2+v}}; v+1\right)$$

where $g(x;v)$ and $G(x;v)$ are the p.d.f. and c.d.f. of the usual Student's- t distribution with v degrees of freedom.

27. Chi-square Distribution

$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}$$

where $x > 0$ if $k = 1$, otherwise $x \geq 0$

$$F(x) = \frac{1}{\Gamma(k/2)} \gamma \left(\frac{k}{2}, \frac{x}{2} \right)$$

28. F Distribution

$$f(x) = \frac{\sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x \, \mathrm{B}\left(\frac{d_1}{2}, \frac{d_2}{2}\right)} \quad (d_1 > 0; d_2 > 0)$$

where $x > 0$ if $d_1 = 1$, otherwise $x \geq 0$

$$F(x) = I_{\frac{d_1 x}{d_1 x + d_2}} \left(\frac{d_1}{2}, \frac{d_2}{2} \right)$$

where $I_x(a, b) = \frac{\mathrm{B}(x; a, b)}{\mathrm{B}(a, b)}$ and $\mathrm{B}(x; a, b) = \int_0^x t^{a-1}(1-t)^{b-1} dt$.

29. Pareto Distribution

$$f(x) = \frac{\alpha x_{\mathrm{m}}^{\alpha}}{x^{\alpha+1}} \quad (x \geq x_{\mathrm{m}}; x_{\mathrm{m}} > 0; \alpha > 0)$$

$$F(x) = 1 - \left(\frac{x_{\mathrm{m}}}{x} \right)^{\alpha}$$